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FACTORISATION OF THE SUM AND DIFFERENCE OF CUBES

The sum and difference of perfect cubes always factorise into the form shown below:

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

1st bracket : **cube root each term** and **keep the same sign**

2nd bracket : square the first term, **change the sign and multiply the terms together**

Square the last term (the sign of the last term is always positive)

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

The sum and difference of cubes always factorises into a binomial and a trinomial. 💡

Example 1: Factorise $8x^3 + y^3$.

$$\begin{aligned} 8x^3 + y^3 \\ = (2x + y)(4x^2 - 2xy + y^2) \end{aligned}$$

Example 2: Factorise $54x^3 - 16$.

$$\begin{aligned} 54x^3 - 16 &\rightarrow \text{always look for a HCF when factorising (2 is common to both terms)} \\ &= 2(27x^3 - 8) \\ &= 2(3x - 2)(9x^2 + 6x + 4) \rightarrow \text{you cannot drop the 2} \end{aligned}$$

Example 3: Factorise $(x - y)^3 - 64x^3$.

$$\begin{aligned} (x - y)^3 - 64x^3 &\rightarrow \text{difference of two perfect cubes} \\ &= [(x - y) - 4x][(x - y)^2 + 4x(x - y) + 16x^2] \\ &= (x - y - 4x)(x^2 - 2xy + y^2 + 4x^2 - 4xy + 16x^2) \rightarrow \text{gather like terms} \\ &= (-3x - y)(21x^2 - 6xy + y^2) \end{aligned}$$



In factorisation of the sum and difference of two cubes the trinomial, (in the second bracket) will never factorise further.



Complete Exercise 3 number 5 on page 240.

A. NUMBER PATTERNS

In Grade 10 you are required to identify number patterns and obtain a general formula for any term in the pattern.

LINEAR NUMBER PATTERNS

In linear number patterns there is a common difference between consecutive terms. This means that the same value is either added to or subtracted from each preceding term.

In a linear number pattern the general formula for the n^{th} term in a pattern is:

$$T_n = dn + c$$

Where: $n \rightarrow$ the position of the term in the sequence (ie: term 1, term 2, ...)

$d \rightarrow$ the common difference between sequential terms ($T_2 - T_1; T_3 - T_2; \dots$)

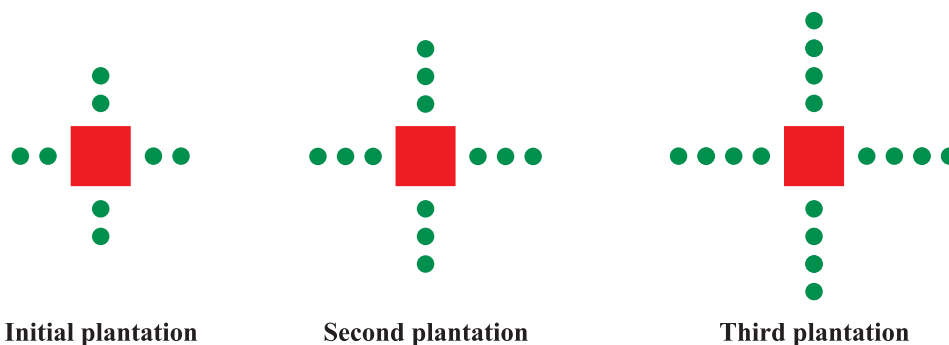
$c \rightarrow$ a constant term

If a number pattern is linear, the difference between any two sequential terms in the pattern will be the same. In other words: $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = T_n - T_{n-1}$

Steps to writing an expression for the n^{th} term of a linear number pattern:

1. Determine if the pattern is linear by checking that: $d = T_2 - T_1 = T_3 - T_2$
2. Substitute d into the general formula: $T_n = dn + c$
3. Substitute a term (usually the first term) into the formula and solve for c .
4. Write down the final expression.

Example 1: Seedlings on a farm are planted around an underground watering system in the following pattern:



- a) How many seedlings will there be in the fourth plantation?
- b) Write a rule for the number of seedlings required for the n^{th} plantation in words. This is known as a conjecture. 💡
- c) Write a mathematical formula for the number of seedlings required for the n^{th} plantation.
- d) How many seedlings will there be in the 12th plantation?
- e) How many plantations will there be when there are more than 88 seedlings?

2. SUBSTITUTION

When using substitution to solve simultaneous equations, you have to make one of the variables the subject of the equation.

You then substitute the solution of that variable into the other equation in order to eliminate one of the variables.

STEPS TO SOLVING SIMULTANEOUS EQUATIONS BY SUBSTITUTION

1. Label the equations as equation 1 and equation 2 respectively.
2. In one equation, make one of the variables the subject of the equation.
3. Substitute into the other equation.
4. Solve for the unknown variables.

Example 1: Solve for x and y if $4x + 8y = 36$ and $2x + 9y = 28$.

$$4x + 8y = 36 \rightarrow \text{equation } \textcircled{1}$$

$$2x + 9y = 28 \rightarrow \text{equation } \textcircled{2}$$

From equation $\textcircled{1}$:

$$4x = 36 - 8y \rightarrow \text{make } x \text{ the subject of the equation}$$

$$\therefore x = 9 - 2y \rightarrow \text{equation } \textcircled{3}, \text{ divide each term by 4 in order to solve for } x$$

substitute $\textcircled{3}$ into $\textcircled{2}$:

$$\therefore 2(9 - 2y) + 9y = 28 \rightarrow \text{substitute } x = 9 - 2y, \text{ for all the } x\text{'s in equation 2}$$

$$\therefore 18 - 4y + 9y = 28$$

$$\therefore 5y = 10$$

$$\therefore y = 2 \rightarrow \text{equation } \textcircled{4}$$

substitute $\textcircled{4}$ into $\textcircled{3}$:

$$\therefore x = 9 - 2(2) \rightarrow y = 2$$

$$\therefore x = 5 \text{ and } y = 2$$

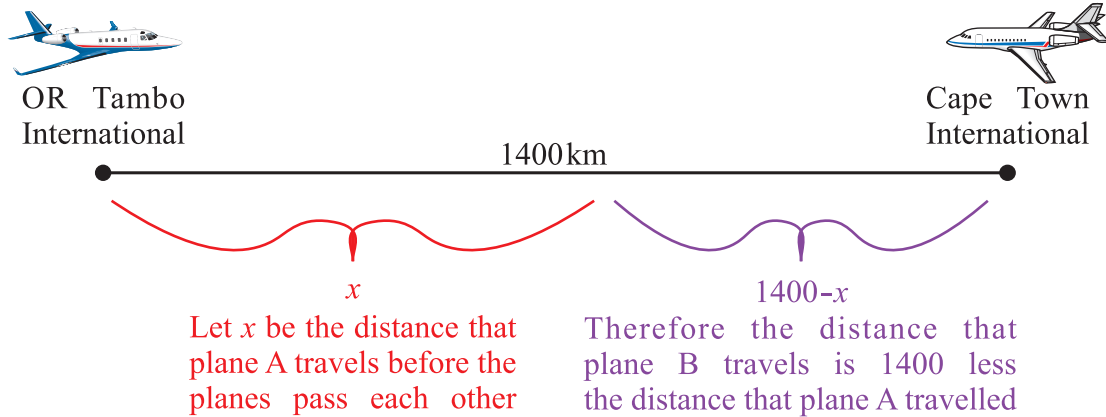


In a test or exam you may be told which method to use when solving simultaneous equations. If you use a different method from the one you are told to use, you will receive no marks.

Example 2:

Plane A takes off from OR Tambo International Airport heading towards Cape Town International Airport. At the same time plane B takes off from Cape Town International Airport, heading for OR Tambo International flying 20 km/h slower than Plane A. The airports are 1400 km apart and the planes meet after 54 minutes. Calculate the distance from OR Tambo International at which they meet and the speed of the planes.

When objects are moving towards each other a diagram can help in drawing up a table.



You cannot work in minutes as the speed is in kilometres per **hour**. Therefore, you must convert minutes to hours: $\frac{54 \text{ minutes}}{60} = 0,9 \text{ hours}$

Situation	Distance	Speed	Time
Plane A	x	$\frac{x}{0,9}$	0,9
Plane B	$1400 - x$	$\frac{1400 - x}{0,9}$	0,9

You can now use the extra information in the question (Plane A travels 20 km/h faster than Plane B) to make an equation.

$$\frac{x}{0,9} = \frac{1400 - x}{0,9} + 20 \rightarrow \text{Plane A flies 20 km/h faster than Plane B}$$

$$\therefore x = 1400 - x + 18 \rightarrow \text{both sides were multiplied by 0,9}$$

$$\therefore 2x = 1418$$

$$\therefore x = 709$$

\therefore The planes will pass each other 709 km from OR Tambo International

$$\therefore \text{Speed of plane A} = \frac{x}{0,9} = \frac{709}{0,9} = 787,78$$

$$\therefore \text{Speed of plane B} = 787,78 - 20 = 767,78$$

\therefore Plane A is flying at 787,78 km/h and plane B is flying at a speed of 767,78 km/h



To convert minutes to hours you divide by 60.

Example 1: If $\sin \theta = \frac{3}{5}$ and $\theta \in [90^\circ; 270^\circ]$ determine the value of $\cos \theta \tan^2 \theta$.

With this type of question you have to draw a diagram in order to solve the problem. 💡

Step 1: determine which quadrant you are in and draw a diagram:

You are given that $\sin \theta = \frac{3}{5}$.

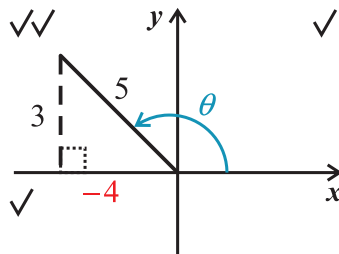
$\frac{3}{5}$ is positive, therefore θ lies in quadrant 1 or 2 as $\sin \theta$ is positive in these quadrants.

Therefore place a tick in the 1st and 2nd quadrant. You now have to use the restriction in the question. You are told that θ is between 90° and 270° . This means that θ is in the 2nd or 3rd quadrant. Therefore place a tick in the 2nd and 3rd quadrant. The quadrant with two ticks is where both constraints are satisfied and this is the quadrant that the terminal arm lies in.

$\sin \theta = \frac{3}{5} \rightarrow \frac{y}{r}$ therefore the y-value is 3 and r is 5.

You can now draw the diagram as shown below:

The value of **-4** in red is calculated in step 2 and then added to the diagram. 💡



Step 2: Calculate the value of the unknown side using Pythagoras: $r^2 = x^2 + y^2$

$$\therefore 5^2 = x^2 + 3^2$$

$$\therefore x^2 = 16$$

$$\therefore x = \pm 4$$

$$\therefore x = -4 \rightarrow \text{negative as you are in the second quadrant}$$

Step 3: Now solve the problem using your diagram.

$$\begin{aligned} \cos \theta \tan^2 \theta &= \left(\frac{-4}{5}\right) \left(\frac{3}{-4}\right)^2 \\ &= \left(\frac{-4}{5}\right) \left(\frac{9}{16}\right) \\ &= -\frac{9}{20} \end{aligned}$$

A common mistake is to write the above answer as:



$$\cos\left(\frac{-4}{5}\right) \tan^2\left(\frac{3}{-4}\right) = -\frac{9}{20}$$

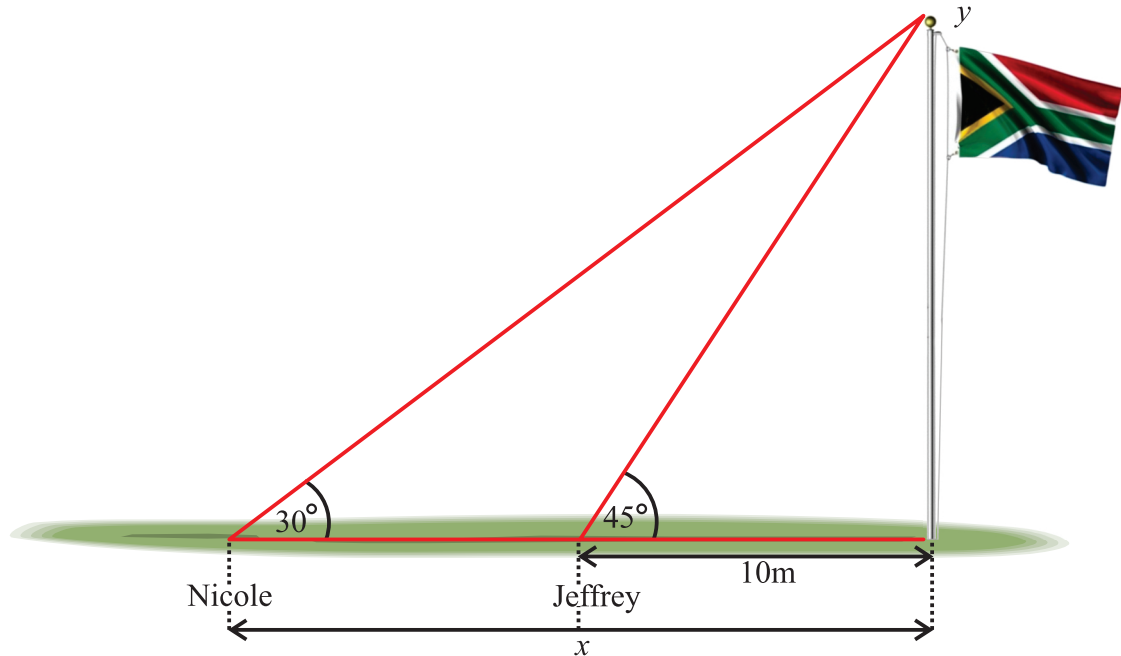
This is **incorrect!** The sides of the triangle can never be written as the angle.



The line drawn in the Cartesian plane is known as the terminal arm. In this question the terminal arm has a length of 5 units and lies in the 2nd quadrant. Remember that the length of a line can never be negative. No matter which quadrant the terminal arm lies in, r is always positive.

Example 3: Nicole and Jeffrey are standing behind each other in a straight line watching a flag blowing in the wind. The angle of elevation of the flag from Jeffrey is 45 degrees and Nicole's angle of elevation to the flag is 30 degrees.

- If Jeffrey is standing 10m from the base of the flagpole, calculate the height of the flag pole.
- How far are Jeffrey and Nicole standing from each other?



- Using the triangle that Jeffrey makes with the pole: let y be the height of the pole.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore \tan(45^\circ) = \frac{y}{10} \rightarrow y \text{ is the height of the pole}$$

$$\therefore y = 10 \tan(45^\circ)$$

$$\therefore y = 10 \text{ m} \rightarrow \text{always put in units}$$

\therefore The height of the flagpole is 10m

- Using the triangle that Nicole makes with the pole:

$$\therefore \tan(30^\circ) = \frac{y}{x} \rightarrow y \text{ is the height of the pole, } x \text{ is the distance Nicole is from the pole}$$

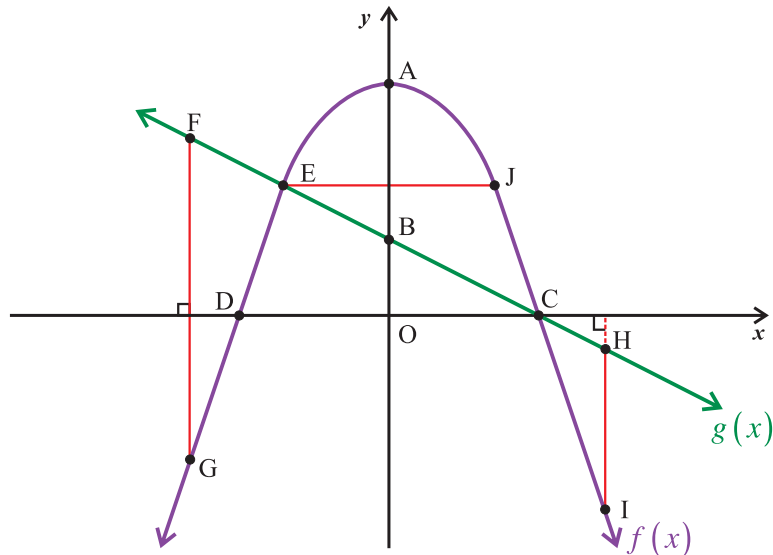
$$\therefore \tan(30^\circ) = \frac{10}{x}$$

$$\therefore x = \frac{10}{\tan(30^\circ)}$$

$$\therefore x = 17,3 \text{ m} \rightarrow \text{this is the distance Nicole is from the base of the pole}$$

The question asked for the distance that Jeffrey and Nicole are from each other

$$\therefore \text{Nicole is standing } 7,3 \text{ m from Jeffrey} \rightarrow 17,3 - 10 = 7,3$$



f) AS EJ is parallel to the x -axis, the y -value of E is the same as the y -value at J.

\therefore The y -value at J is $\frac{15}{2}$

g) The parabola is symmetrical about the y -axis. This means that J is a reflection of E about the y -axis.

\therefore J is the point $\left(\frac{1}{2}; \frac{15}{2}\right) \rightarrow$ the x -value at E is $-\frac{1}{2}$

h) The question is asking for what values of x are the y -values of $f(x) > g(x)$.

This occurs from point E up until point C.

$\therefore f(x) > g(x)$ where $x \in \left(-\frac{1}{2}; 2\right) \rightarrow$ this could also be written as $-\frac{1}{2} < x < 2$

i) The question is asking for what values of x are the y -values of $\frac{f(x)}{g(x)} \geq 0$.

For this to happen $f(x)$ and $g(x)$ both have to be positive or both have to be

negative, as $\frac{'+'}{'+'} \rightarrow '+'$ or $\frac{'-'}{'-'} \rightarrow '+'$

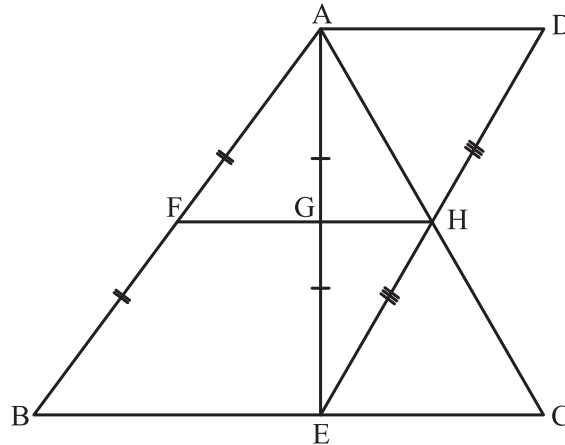
- From points D to C $f(x)$ and $g(x)$ are both positive.
- From point C onwards $f(x)$ and $g(x)$ are both negative.
- It is important to note that at the point where $x = 2$, $g(x) = 0$. Therefore $x \neq 2$ as division by 0 would occur.

$\therefore \frac{f(x)}{g(x)} \geq 0$ where $x \in [-2; \infty)$; $x \neq 2 \rightarrow$ this could also be written as $x \geq -2$; $x \neq 2$



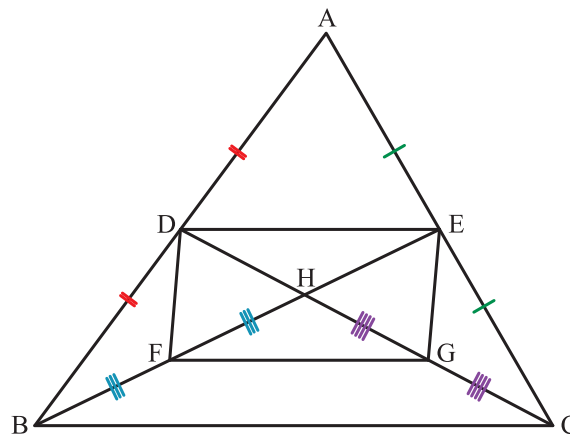
Remember that where a graph lies above the x -axis it is positive and where it lies below the x -axis the graph is negative.

Example 1: In the diagram below, F is the midpoint of AB, G is the midpoint of AE and H is the midpoint of DE. Prove that $AD = CE$.



Statement	Reason
$AF = FB$ and $AG = GE$	given
$\therefore FG \parallel BE \rightarrow$ this means $GH \parallel EC$	midpt. thm.
$\therefore CE = 2GH$	conv. midpt. thm.
$DH = HE$ and $AG = GE$	given
$\therefore AD = 2GH$	midpt. thm.
$\therefore AD = CE$	

Example 2: In the diagram below, D is the midpoint of AB, E is the midpoint of AC, F is the midpoint of BH and G is the midpoint of HC. Prove that DFGH is a parallelogram.



Statement	Reason
$AD = DB$ and $AE = EC$	given
$\therefore BC = 2DE$ and $DE \parallel BC$	midpt. thm.
$BF = FH$ and $CG = GH$	given
$\therefore BC = 2FG$ and $BC \parallel FG$	midpt. thm.
$\therefore DE = FG$ and $DE \parallel FG$	
$\therefore DFGH$ is a parm.	one pair of opp, sides = and //



Complete Exercise 6 on pages 328 and 329.

Example 1: Given the points A(3;-2) and B(-5;4), determine the length of line segment AB.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 3)^2 + [4 - (-2)]^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

Example 2: Given the points A(4;10) and B(-3;-8), determine the length of line segment AB. Give your answer correct to one decimal place.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3 - 4)^2 + (-8 - 10)^2} \\ &= \sqrt{49 + 324} \\ &= \sqrt{373} \\ &= 19,3 \text{ units} \end{aligned}$$

Example 3: Given the points P(-5;6) and Q(-9;y), determine the value(s) of y if PQ = 5 cm.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ \therefore 5 &= \sqrt{[-9 - (-5)]^2 + (y - 6)^2} \\ \therefore 5 &= \sqrt{(-4)^2 + (y - 6)^2} \rightarrow \text{square both sides} \\ \therefore 25 &= (-4)^2 + (y - 6)^2 \\ \therefore 25 &= 16 + y^2 - 12y + 36 \rightarrow \text{dont forget the middle term} \\ \therefore 25 &= y^2 - 12y + 52 \\ \therefore 0 &= y^2 - 12y + 27 \\ \therefore 0 &= (y - 3)(y - 9) \\ \therefore y &= 3 \text{ or } y = 9 \end{aligned}$$

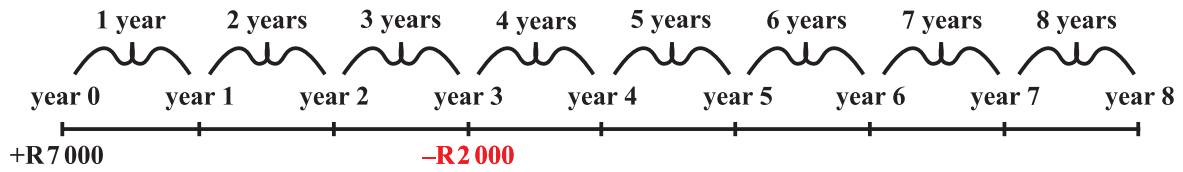


The distance formula can also be written as: $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



Complete Exercise 7 number 1 on page 329.

Example 2: Phindile deposits R7 000 into a savings account which offers an interest rate of 8% p.a. compounded annually. 3 years later she withdraws R2 000. How much will she have in her account at the end of 8 years?



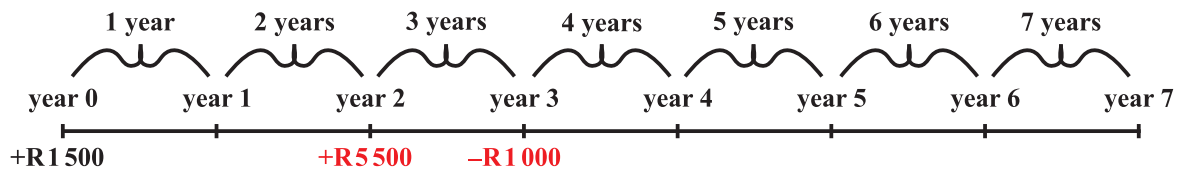
When dealing with a withdrawal you have to remove the withdrawal with its interest for the time that the money was **not** in the account

$$A = 7000(1+0,08)^8 - 2000(1+0,08)^5 \rightarrow \text{R2 000 was not in the account for 5 years}$$

$$= \text{R10 017,86}$$

∴ Phindile will have R10 017,86 in her account after 8 years.

Example 3: Rebecca deposits R1 500 into a savings account which offers an interest rate of 11% p.a. compounded annually. 2 years later she deposits R5 500 into the account and withdraws R1 000 one year later. Determine the amount that Rebecca will have in her account after 7 years.

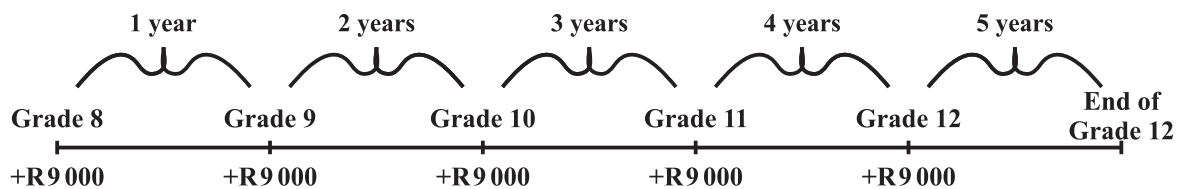


$$A = 1500(1+0,11)^7 + 5500(1+0,11)^5 - 1000(1+0,11)^4$$

$$= \text{R10 863,99}$$

∴ Rebecca will have R10 863,99 in her account after 7 years.

Example 4: Mrs Reddy deposits R9 000 into a savings account at the beginning of each year in order to save for a car for her son. She makes her first deposit when her son starts grade 8 and her last deposit when he starts Grade 12. If the account offers an interest rate of 10% per annum, how much will Mrs Reddy have saved if she buys a car for her son at the end of Grade 12?



$$A = 9000(1+0,1)^5 + 9000(1+0,1)^4 + 9000(1+0,1)^3 + 9000(1+0,1)^2 + 9000(1+0,1)^1$$

$$= \text{R60 440,49}$$

∴ Mrs Reddy will have R60 440,49 saved for her son's car.

D. THE FIVE NUMBER SUMMARY

The different measures of dispersion that you have learnt so far can be represented and analysed graphically on a box and whisker diagram using the Five Number Summary. The five number summary is:

1) **Lowest value in the data set.**

The first of the five number summary, is the lowest (minimum) value in the ordered data set.

2) **Lower Quartile / First Quartile / 25th percentile (Q_1).**

The lower quartile is the median of the lower half of the ordered data set.

3) **Median / Second Quartile / 50th percentile (Q_2).**

The median is the middle value in an ordered set of data. If there is an even number of terms, the median will be the sum of the two terms in the middle divided by two.

4) **Upper Quartile / Third Quartile / 75th percentile (Q_3).**

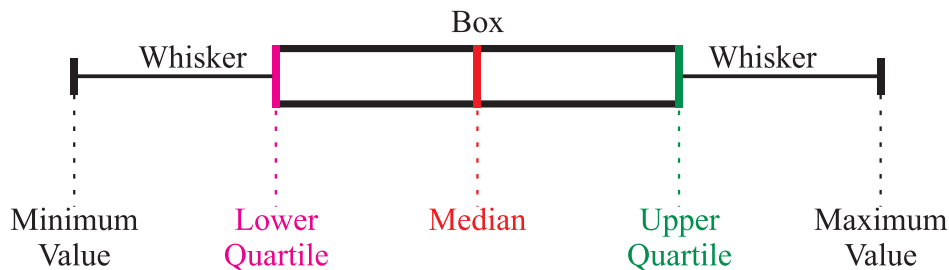
The upper quartile is the median of the upper half of the ordered data set.

5) **Highest value in the data set.**

The last of the five number summary, is the highest (maximum) value in the ordered data set.

BOX AND WHISKER DIAGRAMS

A box and whisker plot is a graphical representation of the Five Number summary. A box and whisker plot can assist in analysing the spread of data about the median.

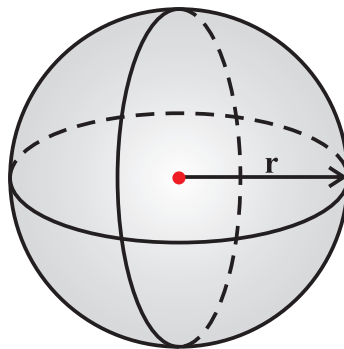


It is important to note the following about the box and whisker diagrams:

- half of the values lie between the minimum value and the **median**
- half of the values lie between the **median** and the maximum value
- half of the values lie between the **lower quartile** and **upper quartile**
- a quarter of the values lie between the minimum value and **lower quartile**
- a quarter of the values lie between the **lower quartile** and **median**
- a quarter of the values lie between the **median** and **upper quartile**
- a quarter of the values lie between the **upper quartile** and maximum value

SURFACE AREA AND VOLUME OF A SPHERE

A sphere is a perfectly symmetrical ball with all points on the surface of the sphere being equidistant from the centre. The formulas for surface area and volume of a sphere are shown below.



Surface Area and Volume of a Sphere:

Surface Area

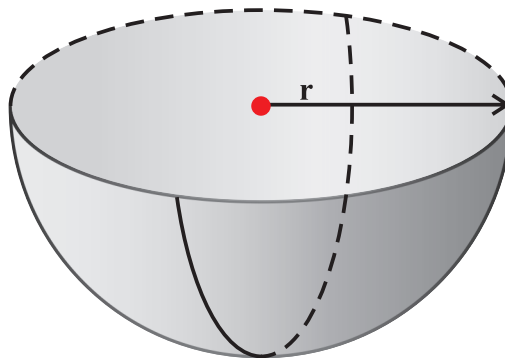
$$SA = 4\pi r^2$$

Volume

$$\text{Volume} = \frac{4}{3}\pi r^3$$

SURFACE AREA AND VOLUME OF A HEMISPHERE

A hemisphere is half a sphere. Therefore, the volume of a hemisphere is simply half that of a sphere. However, if the hemisphere is closed you have to take into account the area of the base of the hemisphere when calculating the surface area of a hemisphere. If the hemisphere is closed or solid the area of the base (πr^2) has to be added.



Surface Area and Volume of a Closed/Solid Hemisphere:

Surface Area

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2 = 3\pi r^2$$

Volume

$$\text{Volume} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$$

Surface Area and Volume of an Open Hemisphere:

Surface Area

$$SA = \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

Volume

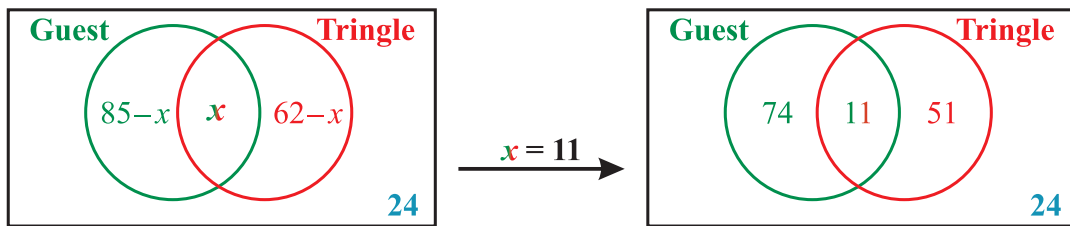
$$\text{Volume} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$$

Example 4: In a marketing survey 160 people were asked if they owned two different brands of denim jeans. The choice was between Guest jeans and Tringle jeans. The results of the survey are given below:

- 85 people owned Guest
- 62 people owned Tringle
- 136 people owned at least one of the brands
- x people owned both brands

- a) Record the above information in a Venn diagram.
 b) How many people own Guest but not Tringle?
 c) What is the probability that a person selected at random owns only one of the brands?
 d) What is the probability that a person selected at random owns one of the brands?

a)



Method:

Step 1: Determine the number of people that wear none of the brands. There are 160 people in the sample space and 136 people who wear at least one of the brands.
 \therefore the number of people who wear non of the brands = $160 - 136 = 24$

Step 2: It is given that **85** people own Guest. However this includes the x number of people that own both Guest and Tringle.
 \therefore Guest only = $85 - x$

Step 3: It is given that **62** people own Tringle. However this includes the x number of people that own both Guest and Tringle.
 \therefore Tringle only = $62 - x$

Step 4: You can now determine the value of x . There were 160 people in the survey.
 $\therefore 85 - x + 62 - x + x + 24 = 160$
 $\therefore x = 11$

b) 74

$$\begin{aligned} \text{c) } P(\text{only Guest or Only Tringle}) &= \frac{74}{160} + \frac{51}{160} \\ &= \frac{25}{32} \\ &= 0,7813 \\ &= 78,13\% \end{aligned}$$

$$\begin{aligned} \text{d) } P(\text{Guest or Tringle}) &= \frac{74}{160} + \frac{51}{160} + \frac{11}{160} \\ &= \frac{17}{20} \\ &= 0,85 \\ &= 85\% \end{aligned}$$



**Complete Exercise 2 number 3 on page 375 and
the Mixed Exercise on pages 375 and 376.**

Mixed Exercise: (Chapter 2 is on pages 9-33)**1) Simplify the following:**

a) $(2x-3)^2$

b) $(2x-y)^2 - (x-2y)(x+2y)$

c) $(3x-5)(2x^2+x-4)$

d) $(8m-3n)(4m+n) - (n-3m)(n+3m)$

e) $(4xy-3)^2 + 24xy$

f) $2xy(3x^2-7xy+10y^2)$

g) $3(x-1)^2 - 2(x+3)(2x-1)$

h) $(-x+2)(x^2-4x+1)$

i) $(x+5)(x^2-3x-9)$

j) $\left(\frac{4}{x}-\frac{3}{y}\right)\left(\frac{4}{x}+\frac{3}{y}\right)$

k) $-x^2\left(\frac{1}{x}+2x\right)\left(\frac{2}{x}-3x\right)$

l) $\left(\frac{x}{4}-3\right)\left(\frac{2x}{3}+5\right)$

m) $2(2x+3)(2x-3) + (2x-1)(x^2-3x+4)$

n) $\frac{3p^2}{q}(4q-2pq) - \frac{p}{5}(5p^2+10p)(5p^2-10p)$

o) $(2x+3)(x-7)(2x-3)(x+7)$

p) $(2a-3b)(8a^3+27b^3)(4a^2+6ab+9b^2)$

Solutions to the Mixed Exercise Number 1 are on page 250

2) Factorise the following completely:

a) xy^2-x

b) $72-8a^2$

c) $6a^2b^3-2ab^2$

d) x^3+16x

e) $\frac{a^2b}{4} - \frac{9b^3}{25}$

f) $2xy+x-10y-5$

g) $x^3-3x^2-4x+12$

h) $3x^2-x-10$

i) $3x^2-9x-xy+3y$

j) $x^2-3x-10$

k) p^3-3p^2-p+3

l) $8x^2+18xy-5y^2$

m) $8p^2+14p-15$

n) $pr-(r+p)x+x^2$

o) $4x^2-2(5x+3)$

p) $xy-xz-y+z$

q) $8p^3-27r^3$

r) $-4x^4y^4-7x^2y^2+2$

s) $36(a-2)^2-9$

t) $3x^3-5x^2+12xy^2-20y^2$

u) $18p^2-24pq+8q^2-32$

v) $x^2(y^2-1)+5x(1-y^2)-6y^2+6$

Solutions to the Mixed Exercise Number 2a and 2b are on page 250 and 2c to 2v are on page 251

Chapter 2 – Manipulation of Algebraic Expressions (Solutions)

Exercise 1 Solutions:

1)

$$\begin{aligned} \text{a)} \quad & 3(x^2 + 3x^3 - 2) \\ & = 3x^2 + 9x^3 - 6 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & 5x(x^2 + y^2 - 3xy) - 3y(xy + 5x^2) \\ & = 5x^3 + 5xy^2 - 15x^2y - 3xy^2 - 15x^2y \\ & = 5x^3 + 2xy^2 - 30x^2y \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & -2(p^2 - 3pq + 8a) - 2(qp - 2a) \\ & = -2p^2 + 6pq - 16a - 2qp + 4a \\ & = -2p^2 + 4pq - 12a \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & 4xy(xy + 2y^2 - 3y) - 2y^2(2x^2 + 4xy + 2x) \\ & = 4x^2y^2 + 8xy^3 - 12xy^2 - 4x^2y^2 - 8xy^3 - 4xy^2 \\ & = -16xy^2 \end{aligned}$$

Exercise 1 number 1 is on page 238

2)

$$\begin{aligned} \text{a)} \quad & (x+3)(x-2) \\ & = x^2 - 2x + 3x - 6 \\ & = x^2 + x - 6 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & (2x+3)(3x-2) \\ & = 6x^2 - 4x + 9x - 6 \\ & = 6x^2 + 5x - 6 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad & (a+b)(a+b) \\ & = a^2 + ab + ab + b^2 \\ & = a^2 + 2ab + b^2 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (x+3)(x-5) \\ & = x^2 - 5x + 3x - 15 \\ & = x^2 - 2x - 15 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & (2x+y)(x-3y) \\ & = 2x^2 - 6xy + xy - 3y^2 \\ & = 2x^2 - 5xy - 3y^2 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & (3a+2b)(5a-3b) \\ & = 15a^2 - 9ab + 10ab - 6b^2 \\ & = 15a^2 + ab - 6b^2 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & (y-3)(y-7) \\ & = y^2 - 7y - 3y + 21 \\ & = y^2 - 10y + 21 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & (2p+q)(p-q) \\ & = 2p^2 - 2pq + pq - q^2 \\ & = 2p^2 - pq - q^2 \end{aligned}$$

$$\begin{aligned} \text{i)} \quad & (2b+3c)(3b-7c) \\ & = 6b^2 - 14bc + 9bc - 21c^2 \\ & = 6b^2 - 5bc - 21c^2 \end{aligned}$$

Exercise 1 number 2 is on page 238

3)

$$\begin{aligned} \text{a)} \quad & (x-y)^2 \\ & = x^2 - 2xy + y^2 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \left(a - \frac{b}{2}\right)^2 \\ & = a^2 - ab + \frac{b^2}{4} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (3x+2y)^2 \\ & = 9x^2 + 12xy + 4y^2 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & \left(\frac{x}{3} - 3\right)^2 \\ & = \frac{x^2}{9} - 2x + 9 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & (7x+2y)^2 \\ & = 49x^2 + 28xy + 4y^2 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad & (x^a + b^y)^2 \\ & = x^{2a} + 2x^a b^y + b^{2y} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & (x^3 + y^2)^2 \\ & = x^6 + 2x^3y^2 + y^4 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & \left(\frac{x^2}{3} - \frac{3y^3}{2}\right)^2 \\ & = \frac{x^4}{9} - x^2y^3 + \frac{9y^6}{4} \end{aligned}$$

Exercise 1 number 3 is on page 238